

Mathematics Classroom Studies – Multiple windows (lenses) and perspectives

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An Institute of

Overview

- **Background**

TIMSS Video Studies – 1995 & 1999

- **The Early Stages of Mathematics Classroom Studies in Singapore**

Kassel Project & Study of Grade 5 Mathematics Lessons

- **The Learner's Perspective Study**

- **Traditional Teaching & East Asian Countries**

The CORE 2 Study in Singapore

What's next!

Background

TIMSS Video Study 1995

TIMSS Video Study 1999

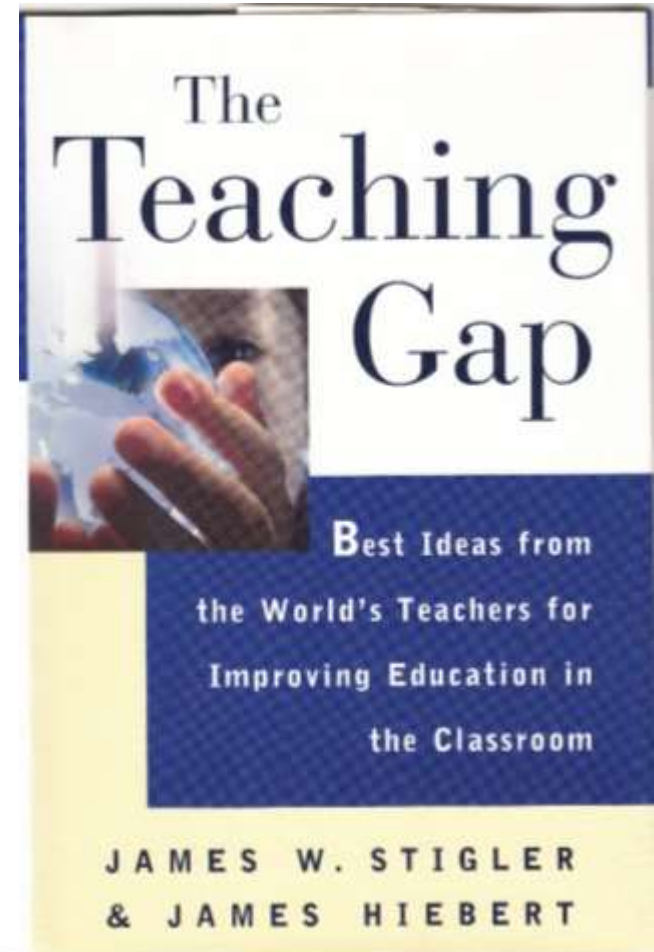
TIMSS Video Study 1995

Germany

Japan

United States

- Eight-grade mathematics classes
- National samples of teachers participated
- One lesson per teacher was recorded
- Lessons recorded: 100 in Germany, 50 in Japan & 81 in the United States



A significant finding of the 1995 Video Study

Teaching is a cultural activity

(Stigler & Hiebert, 1999, p. 11)

“To put it simply, we are amazed at how much teaching varied across cultures and how little it varied within cultures”

“We learn how to teach indirectly, through years of participation in classroom life, and we are largely unaware of some of the most widespread attributes of teaching in our own culture”.

Big Picture Perspective

Patterns of Teaching in Germany, Japan and the U.S.

(The Teaching Gap, pp. 78-81)

The German Pattern [4 activities]	The Japanese Pattern [5 activities]	The U.S. Pattern [4 activities]
Reviewing previous material	Reviewing the previous lesson	Reviewing previous material
Presenting the topic and problems for the day	Presenting the problem for the day	Demonstrating how to solve problems for the day
Developing procedures to solve the problem/s	Students working individually or in groups	Practicing
Practicing	Discussing solution methods	Correcting seatwork and assigning homework
	Highlighting and summarizing the major points	

TIMSS 1999 Video Study

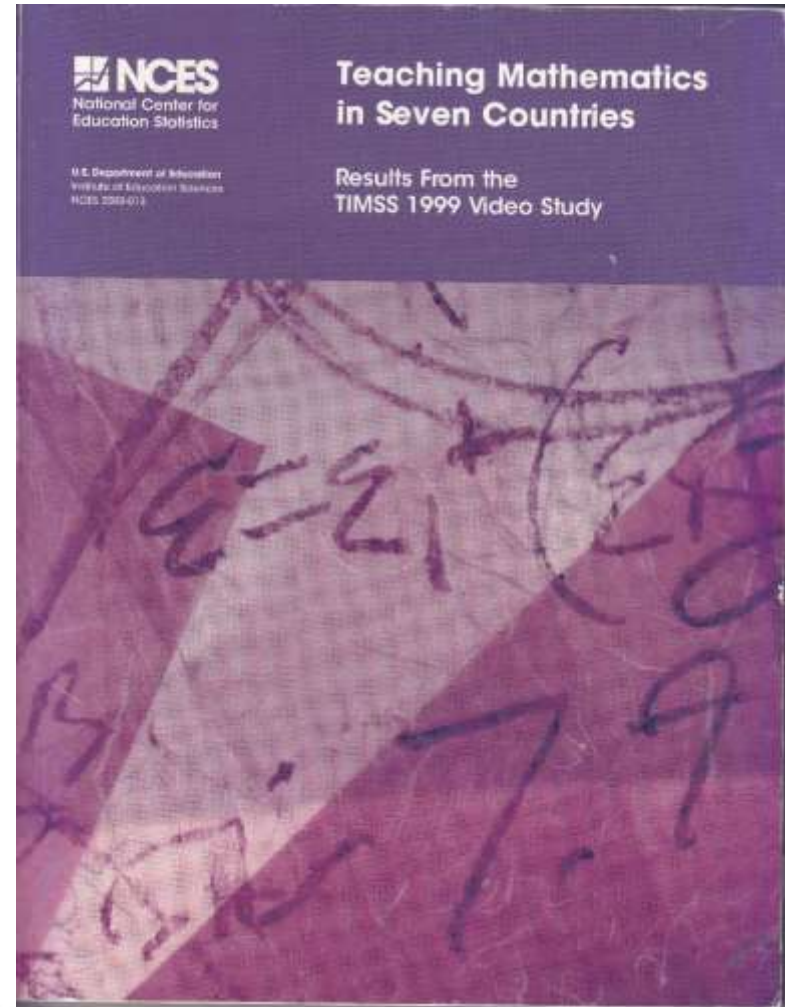
7 countries were involved - Australia, Czech Republic, Hong Kong SAR, Japan, Netherlands, Switzerland, United States

Revision of coding schemes

Recognized the limitation of big picture perspectives and wide-angle lens findings

Advocated the use of close-up lens for meaningful interpretations of findings

Significant extension – for readers to digest the contents of the report/s and engage in more nuanced international discussions of mathematics teaching



The Early Stages of Mathematics Classroom Studies in Singapore

The 1990s

Good performance of Singapore students in TIMSS 1995 and also subsequent TIMSS has drawn a lot of attention to the teaching and learning of mathematics in Singapore schools. Educators in Singapore, themselves have also become more curious of activities in their mathematics classrooms.

Two studies amongst the few that may be considered to be amongst the first to document activities in mathematics classrooms in Singapore were:

- **Kassel Project (1995 - 1996)** (Kaur & Yap, 1997)

An international comparative project on the Teaching and Learning of Mathematics helmed of the Centre for Innovation in Mathematics Teaching at the University of Exeter. It was Prof Gabriele Kaiser who initiated Singapore's participation in the project.

- **A Study of Grade 5 Mathematics Lessons (1998-1999)** (Chang, Kaur, Koay & Lee, 2001)

A small scale study to investigate the pedagogical practices of grade 5 mathematics teachers in Singapore.

Kassel Project (1995 – 1996) (Kaur & Yap, 1997)

As part of the Kassel Project, 21 Grade 8 mathematics lessons in 1995 and 22 Grade 9 mathematics lessons in 1996 were observed by Prof Kaur and Dr Yap at the NIE. **Lesson Review Sheets** were used to document observations

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APPENDIX K
KASSEL PROJECT
Lesson Review Sheet

Form: Head Teacher, Other

Date: / /

Observer: Topics: Lesson Reference:

Start time: End time:

Time	Topic	TP/Pupil or Group	Pupil/T	Classworking	Other Notes
5					
10					
15					
20					
25					
30					
35					
40					
45					
50					
55					
60					
65					
70					

Board work
 Homework
 Visual Aids
 Coursework
 HOMEWORK
 Set
 Plan of classroom
 Activities

EPV: Board work Pupil work W: Workbooks T: Textbook V: Visual Aids P: Paper
 S: Spreadsheets Q: Quizzes A: Assess H: Homework D: Diagrams I: Illustrations M: Making
 Calculators used Scientific Calculators used Graphing Calculators used
 Handwritten notes Printed notes Other notes

GMT/Kassel/Lesson Review Sheet/High 1

Further Details

Board work White Black

State Name / Duration of Lesson

	Teacher	Enthusiasm	Flow	Pace			
Textbook	Lesson structure	1	4	3	2	1	
Workbooks	Mathematical approach	1	4	3	2	1	
	Mathematical correctness	1	4	3	2	1	
	Mathematical progress	1	4	3	2	1	
Visual Aids	Clarity of instructions	1	4	3	2	1	
	Authority	5	4	3	2	1	
	Enthusiasm	5	4	3	2	1	
Coursework	Relationship with pupils	5	4	3	2	1	
	No. present	100	90	80	70	60	
	Class	Quiet	1	4	3	2	1
HOMEWORK	Handwriting	5	4	3	2	1	Lazy
	Happy	5	4	3	2	1	Unhappy
	No Movement	5	4	3	2	1	Class
Set	Questioning	5	4	3	2	1	Unprompting
	Responsive	5	4	3	2	1	Unresponsive
	Understanding	5	4	3	2	1	Confused

Plan of classroom Activities:

Brief Summary

Shared Vocabulary

A glossary of terms was created by the two researchers who observed the lessons to describe the lessons. The glossary also helped readers make sense of the lesson narratives.

Term	Explanation
Teacher Exposition	Teacher presents knowledge by telling and explaining
Teacher Demonstration	Teacher works solution to a task highlighting procedure and explaining how the procedure is used
Deductive Questioning	Teacher asks a sequence of questions which guide pupils to form ideas by reasoning and drawing on prior knowledge
...	
Whole Class Discussion	Teacher structures the flow of the interaction and directs students' involvement and participation; teacher is responsible to ensure that there is a central focus of discussion and that questions keep coming back to the key issue/s.
...	
Direct questions	Questions which call for recall of knowledge (facts / algorithms)
...	
Seatwork (individual / pair)	Pupils do mathematical tasks in class on their own / in pairs.

**Kassel Project Data
21 Grade 8 Mathematics
Lessons (7 Schools)
Observed in 1995**

Lesson Narratives

Summary of Lesson Observations

Lesson Observation (1)

Date: 15-4-96
 School code : YO 1
 Teacher code : 1
 Class: Year 8 (Express)
 Duration: 30 minutes
 No. of pupils : boys (15), girls (19), Total (34)

Topic: Linear Inequalities

Teaching Approach

Teacher introduced the inequality signs to the class on the chalkboard. Used the number line to show values of x which satisfied the inequality concentrating on integer answers for inequalities such as $3 < x < 5$, $2 < x < 6$, etc. Lesson was systematically developed with appropriate and carefully chosen examples. Individual pupils were questioned during the lesson to monitor progress. Lesson was highly structured and well organised. Teacher was strict with pupils about their conduct and work (e.g. stand up straight when answering questions, use ruler to draw lines in exercise book, fill out the content page of their exercise books before beginning to do their assignment for the day). Assigned pupils soakwork 5 minutes before the end of the lesson, walked around the class to supervise pupils at work. When the time was up for the lesson pupils were told to complete their assignment at home and hand it in for marking during the next lesson.

teacher exposition, whole class instruction, questioning (individual), direct questions, soakwork (individual, supervised), homework (marked).

Resources / Teaching Aids Used

chalkboard, nerve rule, textbook.

Homework

Reviewed : No

Set : Yes

Teacher Characteristics

Class Characteristics

	Excellent	Poor		Excellent	Poor
Lesson structure	X 4 3 2 1		Quiet	X 4 3 2 1	Noisy
Mathematical approach	X 4 3 2 1		Attentive	X 4 3 2 1	Inattentive
Mathematical correctness	X 4 3 2 1		Happy	5 4 X 2 1	Unhappy
Mathematical progress	5 X 3 2 1		No Movement	X 4 3 2 1	Chaos
Clarity of instructions	X 4 3 2 1		Questioning	5 4 3 X 1	Unquestioning
Authority	X 4 3 2 1		Responsive	5 X 3 2 1	Unresponsive
Enthusiasm	5 4 X 3 1		Understanding	5 X 3 2 1	Confused
Relationship with pupils	5 4 X 2 1				

Lesson Observation (2)

Date: 15-4-96
 School code : YO 1
 Teacher code : 2
 Class: Year 8 (Normal)
 Duration: 25 minutes
 No. of pupils : boys (17), girls (13), Total (30)

Topic: Quadratic Factorisation

Teaching Approach

Teacher reviewed the three methods of factorisation (HCF, difference of 2 squares and grouping) taught during the previous lessons. A procedural approach was adopted by the teacher to teach the pupils how to factorise quadratic expressions. As teacher factorised expressions on the OHT she reinforced and checked if pupils followed the procedure by questioning the whole class and occasionally individual pupils. The lesson ended with pupils being assigned homework. Pupils had to copy the 5 expressions to factorise at home from the OHT. Pupils were told to hand in their homework during the next lesson for marking.

teacher exposition, demonstration, whole class instruction, questioning (individual, class), direct questions, homework (marked).

Resources / Teaching Aids Used

OHT, chalkboard, textbook.

Homework

Reviewed : three methods of factorisation (HCF, diff of 2 squares, grouping)

Set : Yes

Teacher Characteristics

Class Characteristics

	Excellent	Poor		Excellent	Poor
Lesson structure	5 X 3 2 1		Quiet	5 X 3 2 1	Noisy
Mathematical approach	5 X 3 2 1		Attentive	X 4 3 2 1	Inattentive
Mathematical correctness	5 X 3 2 1		Happy	5 X 3 2 1	Unhappy
Mathematical progress	5 X 3 2 1		No Movement	X 4 3 2 1	Chaos
Clarity of instructions	5 X 3 2 1		Questioning	5 4 3 X 1	Unquestioning
Authority	X 4 3 2 1		Responsive	X 4 3 2 1	Unresponsive
Enthusiasm	5 X 3 2 1		Understanding	5 X 3 2 1	Confused
Relationship with pupils	5 X 3 2 1				

Coding and Descriptive Statistics

Teacher Characteristics

Lesson No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Lesson Structure	5	4	5	3	3	4	5	4	5	5	5	3	4
Mathematical approach	5	4	4	3	4	3	5	4	3	4	3	3	4
Mathematical correctness	5	4	4	3	3	4	5	4	4	3	4	4	4
Mathematical progress	4	4	4	4	3	3	5	4	4	4	3	3	4
Clarity of instructions	5	4	4	4	4	5	5	5	4	4	4	4	4
Authority	5	5	4	5	4	5	5	5	4	5	5	4	5
Enthusiasm	3	4	4	3	4	4	4	4	3	5	3	3	4
Relationship with pupils	3	4	5	5	5	4	4	5	4	4	4	4	4

Lesson No.	14	15	16	17	18	19	20	21	Total	Mean
Lesson Structure	4	4	3	4	5	5	5	5	90	4.3
Mathematical approach	3	4	3	5	4	5	5	5	83	4.0
Mathematical correctness	3	3	3	5	3	4	4	4	80	3.8
Mathematical progress	4	4	4	4	4	4	4	4	81	3.9
Clarity of instructions	4	5	5	5	4	4	4	5	92	4.4
Authority	4	5	5	5	3	5	5	5	98	4.7
Enthusiasm	4	4	3	4	4	5	5	5	82	3.9
Relationship with pupils	4	5	3	5	4	5	5	4	90	4.3

Class Characteristics

Lesson No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Quiet-Noisy	5	4	4	3	3	5	5	5	5	5	5	3	4
Attentive-Inattentive	5	5	4	4	4	5	5	4	5	5	5	5	4
Happy-Unhappy	3	4	4	4	4	4	3	5	3	4	3	3	4
No Movement-Chaos	5	5	4	3	3	5	5	5	5	5	5	5	5
Questioning- Unquestioning	2	2	3	3	4	1	3	4	4	3	1	2	4
Responsive- Unresponsive	4	5	4	4	4	4	3	4	4	3	3	3	4
Understanding-Confused	4	4	4	4	3	4	4	4	4	4	3	4	4

Lesson No.	14	15	16	17	18	19	20	21	Total	Mean
Quiet-Noisy	5	4	4	4	4	5	5	4	91	4.3
Attentive-Inattentive	4	5	4	4	5	5	5	4	96	4.6
Happy-Unhappy	3	4	3	4	4	4	4	4	78	3.7
No Movement-Chaos	5	5	4	4	5	5	5	5	98	4.7
Questioning- Unquestioning	3	3	3	2	2	3	3	3	58	2.8
Responsive- Unresponsive	4	4	4	3	4	4	4	4	80	3.8
Understanding-Confused	4	4	3	3	4	4	4	4	80	3.8

Scoring scale:
5 (Excellent) ... 1 (Poor)

Findings (Wide-Angle Lens)

Teachers:

task-oriented; presented knowledge by telling and explaining; demonstrated how to solve mathematical problems(step by step, placed more emphasis on procedures, answers and accuracy than on concepts and processes);

enthusiastic about their teaching; had high expectations of their pupils; handled the mathematics confidently; gave instructions that were candid and clear; lessons were highly structured with specific achievable objectives;

almost always assigned homework and graded it.

used the chalkboard, textbook and overhead projector.

Pupils:

quiet, appeared attentive (even though at times teacher talk was too lengthy to sustain pupil attention); looked happy; seldom volunteered responses or raised doubts; task-oriented and receptive to the teaching.

A Study of Grade 5 Mathematics Lessons (1998-1999) (Chang, Kaur, Koay & Lee, 2001)

- This study may be considered to be the first that video-recorded mathematics lessons to investigate the **pedagogical practices** of grade 5 mathematics teachers following two initiatives, namely the **Infusion of Thinking Skills** and the use of **Information Technology** in Singapore schools.
- 4 Grade 5 teachers from two schools (2 from each school) with distinctively different pupil profiles participated in the study.
- Altogether 5 one-hour lessons were recorded.
- Teachers were also interviewed about their lessons.

Analysis of the data

A wide-angle lens was adopted for the analysis of the lessons by the researchers as they were mainly interested in locating at the macro-level

- similarities and differences in the lessons in the two schools
- the impact of the initiatives (Thinking Skills & IT) on the pedagogy of the teachers.

Findings (Wide-Angle Lens)

Similarities and Differences:

In both the schools:

- Lessons were mainly teacher-directed; two-thirds of the lesson time was devoted to Teacher Talk while a third was utilized for Pupil-Work (individually or group-work).
- Pupil Talk comprised answering teacher-initiated questions or seeking clarifications.
- Tasks enacted during the lessons mainly encouraged comprehension and application of knowledge.
- Classwork and homework focused mainly on development of skills, use of knowledge to complete routine tasks and prepare for examinations.

In school A pupils were also provided with enrichment activities but the activities were not tailored to enhance any specific thinking strategies or skills.

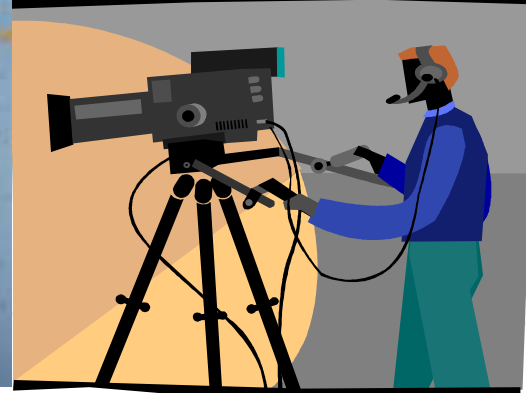
Impact of Initiatives: In both the schools:

- Lessons were teacher directed with little or no evidence of activities to engage pupils in thinking or development of any thinking strategies.
- Infusion of technology in the lessons was also not evident.

The Learner's Perspective Study in Singapore

The Learner's Perspective Study (LPS)

- Singapore joined the LPS in 2004
- The LPS is well known across the world and colleagues participating in it have made numerous presentations in the past ICMEs
- In Singapore, the main objectives of the study were:
 - to document practices of competent mathematics teachers in grade 8 mathematics classrooms,
 - to study from the perspectives of students the roles of the textbook and homework and what constitutes good mathematics lessons, and
 - to identify common classroom pedagogies from the perspectives of both teachers and students that enhance the teaching and learning of mathematics.



Participants of the LPS (Singapore)

3 mathematics teachers recognized by their local communities for 'teaching competence'

- T1 (School 1) – female with 21 years of teaching experience.
- T2 (School 2) – female with 27 years of teaching experience.
- T3 (School 3) – male with 15 years of teaching experience.

The three classes of grade 8 students the teachers taught.

- T1 taught a class of 37 students of average to high ability.
- T2 taught a class of 40 students of average to high ability.
- T3 taught a class of 40 students of low to average ability.

LPS (Singapore)
Selected Data
&
Findings
The insider's perspective

Instructional Approaches (Seah, Kaur & Low, 2006; Kaur, 2009)

- The video records of the ten-lesson sequence for each of the teachers were the main source of the data analyzed.
- Grounded theory approach was adopted. Six categories of instructional format were derived for coding the lessons.

Category	Characteristics
Whole-class demonstration [D]	Whole class mathematics instruction that aimed to develop students' understanding of mathematical concepts and skills
Seatwork [S]	Students were assigned questions to work on either individually or in groups at their desks
Whole-class review of student work [R]	Teachers primary focus was to review the work done by students or the task assigned to them
Miscellaneous [M]	A catch-all category during which the class was involved in managerial and administrative activities
Group quiz [Q]	Found in lessons of T2, students solved tasks in groups in a competitive manner
Test [T]	Found only in lessons of T1 and T3

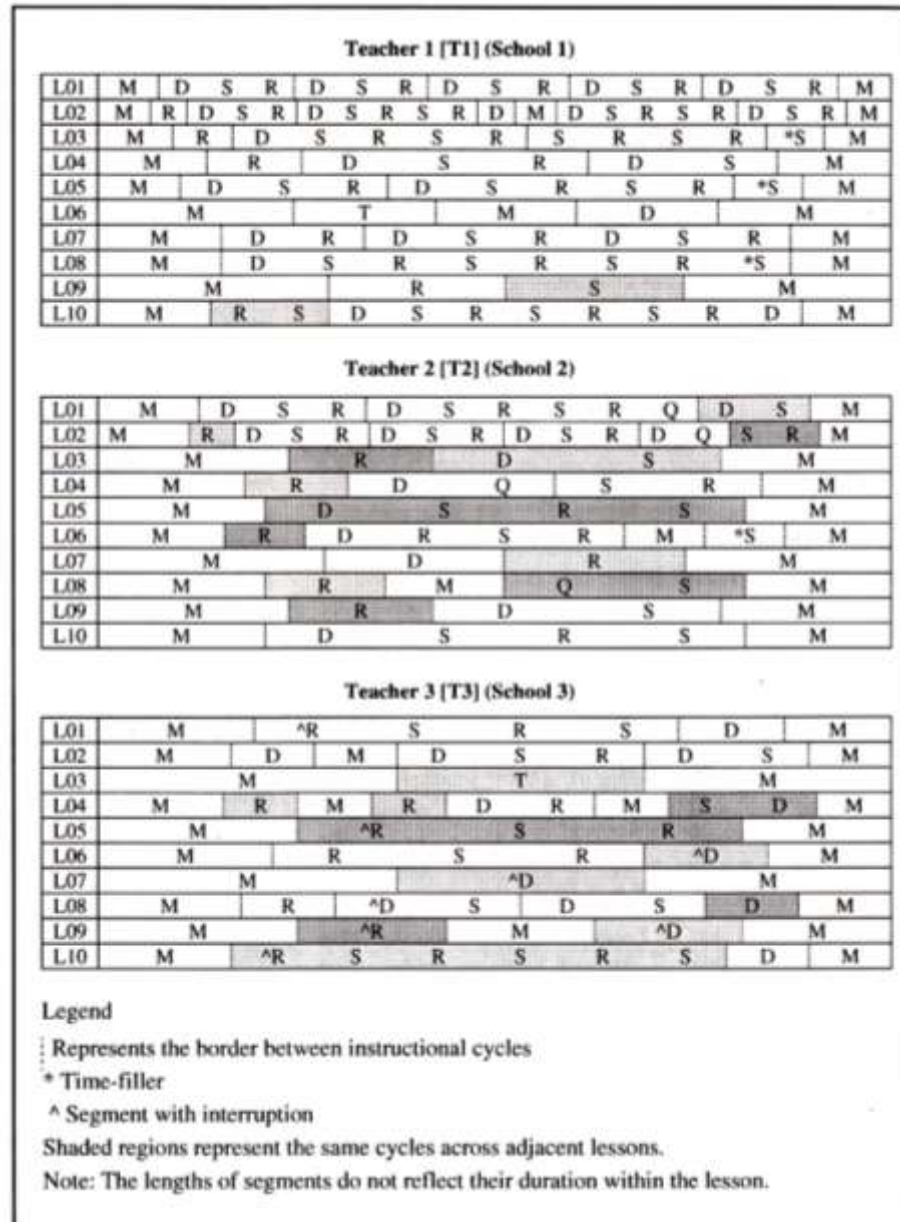
Table 1 Analysis of lesson structure with mathematical content of T2

Lesson no.	Activity segment code	Mathematical content	Instructional objective	Instructional cycle no.
1	[D]	Worked example: $(3x + 2y)^2 - 6x - 4y$	Factorisation by grouping	1
	[S]	Practice task: $2x + 4y - 3(x + 2y)^2$		
	[R]	Student wrote answers for practice task on board		
1	[D]	Worked examples: $x^2 - 9, y^2 - 1/16, 9y^2 - 4z^2$	Factorisation of expression in the form of difference of two squares	2
	[S]	Practice tasks: $a^2x^2 - 16y^2, 50x^2 - 2p^2$		
	[R]	Teacher and students worked out practice tasks on board		
	[S]	Practice tasks: $18m^2 - 8n^4(x - 1)^2 - (2x + 3)^2$		
	[R]	Teacher and students worked out practice tasks on board		
	[Q]	Quiz tasks $4x^2 - 25$ $121 - 36x^2$ $49x^2 - 1$ $\pi R^2 - \pi r^2$		
2	[R]	Reviewed solutions of $6p^4 - 24q^2$ $32xy^4 - 2x^5$ $16n^2 + 8ne + e^2$ $49y^2 + 42yz + 9z^2$ $9f^2 + 24fg + 16g^2$	Factorisation of expressions by grouping and difference of two squares	1

Source of Table: (Kaur, 2009, p.337)

Source of Figure:
Kaur, 2009, p. 338.

Structural patterns of the
lesson sequences of T1, T2
and T3



Duration of Lesson segments

Activity segments	T1		T2		T3	
	Total duration (min)	Total no. of instances	Total duration (min)	Total no. of instances	Total duration (min)	Total no. of instances
Demonstration [D]	111	22	129	14	142	13
Seatwork [S]	217	33	153	18	23	12
Review of student work [R]	135	33	143	19	155	16
Group quiz [Q]	–	–	75	4	–	–
Test [T]	38	1	–	–	23	1
Miscellaneous [M]	32	22	34	22	68	24

Source of Table: Kaur, 2009, p. 339

Nature of classroom talk

Whole class demonstration

- Occupied more than 20% (less than 33%) of the total class time in all the three classrooms.
- Teachers played the most active role in expounding mathematical concepts and problem-solving skills mainly through the use of examples both in the form of concepts and mathematical problems as their teaching tool.
- The examples used were carefully selected on the basis of systematic increase in complexity.

Express the following in the form 10^n .

(1) $10^{12} \times 10^3$

(2) $10^{19} \times 10^{-7}$

(3) $10^{-4} \times 10^{-5}$

Express the following in the form 10^n .

(4) $10^{-6} \div 10^7$

(5) $\frac{10^{-6} \times 10^{-7}}{10^{-14} \times 10^2}$

$10^{-14} \times 10^2$

(6) $\frac{10^{-3} \times 10^{15}}{10^{-7} \times 10^{-28}}$

$10^{-7} \times 10^{-28}$

Whole class demonstration – Discourse pattern

Initiation – Response – Feedback

Episode 1 [T3-L07]

This episode shows T3 demonstrating to the whole class the relationship between an angle and the ratio of the lengths of the arms of a right angled triangle, in particular as the hypotenuse is the longest side of a right angled triangle, the ratio of the opposite side divided by the hypotenuse or the adjacent side divided by the hypotenuse is always less than one.

T: Okay you can see that if I change the angle here, alright the ratio will change accordingly. But can you tell me, is it possible for this ratio to be more than one? Think. Just now, I showed you the tangent right you can go to one you can go beyond one. But is this, is it possible for this ratio to be more than one? Look at this ... look look at this ratio here it's opposite over hypotenuse. Look at this one. Is it possible to go beyond one?

S1: No.

T: What is the reason? You must you must... know the reason why. Why cannot go beyond one? The opposite cannot be longer than the?

Ss: Hypotenuse.

T: Hypotenuse. Very good. Okay this show that you are thinking. Alright? The hypotenuse is always the... long er... the longest. That's why the value is always the biggest. So as long as the denominator is always larger than the nu er... deno er... denominator is... larger than the nominator so... numerator so it will not more be more than one. Agree or not? See I can show it to you ah. [walks to desk] ...

S2: Yes.

T: So okay. [plays around with triangle]

Whole-class review of student work

- This segment followed independent student work (both homework and classwork), group student work or the quiz.
- It occupied about one-quarter of the class time of T1 and of T2, while nearly two-fifths of the class time of T3.
- Although it appeared to be teacher-dominated in form, the main source of content was students' work – mistakes, presentation of solutions and multiple solutions to a problem.
- The discourse was built around student work to learn from mistakes.
- The goal was not the final correct answers but rather to correct the misconceptions/mistakes students made while working through the steps needed to arrive at the final answer.

Episode 3 [T2-L01]

In this episode the teacher is reviewing Harry's (a student) solution and drawing the attention of the whole class to an error in it.

As part of the quiz in class during the lesson Harry was asked to simplify $2x + 4y - 3(x + 2y)^2$ on the board. Harry wrote:

$$\begin{aligned}2x + 4y - 3(x + 2y)^2 &= 2(x + 2y) - 3(x + 2y)(x + 2y) \\ &= (x + 2y) [(x + 2y) + 2 - 3] \\ &= (x + 2y) [x + 2y - 1] \\ &= (x + 2y) (x + 2y - 1)\end{aligned}$$

T: Okay, all of you, have a look at the first one first. ...Here. All of you look here... Alright. Harry grouped these two, take out the common factor two, which is correct, right, this part. And because of the square, he put it like that, so that he can see the common factor. So this one is common. Okay. Right, you take out the common factor, but what is remaining in the first... term, in the bracket. First term in the bracket, it should...should it be this?

Ss: No.

T: X plus 2 Y, Harry? You already taken out this. So what is the remaining term?

S1: Two.

T: Two. It's only two, right. Can you see? Alright. And you have taken out this one, right. So what is remaining?

Ss: Three.

T: It should be three, and then...this one, isn't it? You see. Okay. What about here? Alright, you have taken out this. You have taken out this. So you should have two inside. But why is it the first term is X plus two Y? Okay. Can you try it again?

Findings

Wide-angle lens

The Pattern

- Set the stage / bridge the link
- Present a concept/ procedure show how to work a problem
- Seatwork
- Correct seatwork

Lessons appear to be

- teacher centred;
- mainly comprising teacher exposition coupled with student practice (could be misrepresented as 'drill and practice').

Close-up lens

- Instructional cycles were highly structured comprising combinations of D, S and R.
- Specific instructional objectives guided each instructional cycle, with subsequent cycles building on the knowledge;
- Carefully selected examples that systematically varied in complexity from low to high were used during whole class demonstration;
- There was active monitoring of student's understanding during seatwork (teachers moved from desk to desk guiding those with difficulties and selecting appropriate student work for subsequent whole class review and discussion);
- Student understanding of knowledge expounded during whole class demonstration was reinforced by detailed review of student work done in class or as homework; and
- Lessons were both teacher and student centred.

Students' Perceptions of their Teachers' Teaching

A distinguishing feature of the LPS, is the exploration of learner practices using post-lesson video-stimulated interviews.

The interviews of the “focus students” comprised of two parts. The first part was based on the video-record of the lesson for which they were the focus students. The second part was stimulated by several prompts, two of which were as follows:

- ***Would you describe that lesson as a good one for you?***
- ***What has to happen for you to feel that a lesson was a “good” lesson?***

A total of 59 students were interviewed, 19 from the class of T1, 20 from the class of T2 and 20 from the class of T3. Responses from 57 students to the above two prompts were analysed to establish students' perspectives of good mathematics teaching.

Students' Perceptions (Kaur, 2008, 2009)

- The interview transcripts of 57 students from three classes of the three competent teachers were the source of the data analyzed.
- Grounded theory approach was adopted. Three categories and 12 sub-categories were derived for coding the interview transcripts.

Instructional Practice	Sub-category
Exposition (Whole Class Instruction)	<p>EC - teacher explains / explains clearly</p> <p>D - teacher demonstrates a procedure, “teaches the method” or shows using manipulatives concepts/relationships</p> <p>NK - teacher introduces new knowledge</p> <p>GI - teacher gives instructions (assigning homework / how work should be done / when work should be handed in for grading, etc.)</p> <p>RE - teacher uses real-life examples during instruction</p>
Seatwork	<p>IW - students working individually on tasks assigned by teacher or making / copying notes</p> <p>GW - students working in groups</p> <p>M - material used as part of instruction (worksheet or any other print resource)</p>
Review and Feedback	<p>PK - teacher reviews prior knowledge</p> <p>SP - teacher uses student’s presentation or work to give feedback for in class work or homework</p> <p>IF - teacher giving feedback to individuals during lesson</p> <p>GA - teacher giving feedback to students through grading of their written assignments</p>

Findings: Close-up lens

Analysis of the interview transcripts revealed that students deemed a mathematics lesson as a good one when some of the following characteristics were present.

Teacher

- explained clearly the concepts and steps of procedures,
- made complex knowledge easily assimilated through demonstrations, use of manipulatives, real life examples
- reviewed past knowledge
- introduced new knowledge
- used student work/group presentations to give feedback to individuals or the whole class
- gave clear instructions, related to mathematical activities for in class and after class work
- provided interesting activities for students to work on individually or in small groups
- provided sufficient practice tasks for preparation towards examinations

Perspectives of Good Mathematics Teaching

By **juxtaposing** the findings of the **teachers instructional approaches** and **students' perceptions of good mathematics teaching** by their teachers it is hypothesised that good mathematics teaching in the three grade eight classrooms comprised of three main segments:

Whole-class demonstration (exposition)

Teacher

- explained clearly the concepts and steps of procedures,
- made complex knowledge easily assimilated through demonstrations, use of manipulatives, real life examples
- introduced new knowledge

Seatwork

Teacher

- gave clear instructions, related to mathematical activities for in class and after class work
- provided interesting activities for students to work on individually or in small groups
- provided sufficient practice tasks for preparation towards examinations

Review and feedback

Teacher

- reviewed past knowledge
- used student work/group presentations to give feedback to individuals or the whole class

Traditional Teaching & East Asian Countries

“One common stereotype of East Asian pedagogy is that it is characterized by ‘traditional’ forms of instruction and that this is a major part of the explanation of why East Asian students have done so well in international assessments like TIMSS and PISA” (Hogan et al., 2013, p.63).

Reference:

Hogan, D. et al. (2013). Assessment and the logic of instructional practice in Secondary 3 English and mathematics classrooms in Singapore. *Review of Education*, 1, 57-106.

Leung has noted that mathematics teaching in East Asia, is 'predominantly content orientated and exam driven. Instruction is very much teacher dominated and student involvement minimal'. Teaching is 'usually conducted in whole group settings, with relatively large class sizes'. There is 'virtually no group work or activities, and memorization of mathematics is stressed' and 'students are required to learn by rote'. Students are 'required to engage in ample practice of mathematical skills, mostly without thorough understanding' (2001, pp. 35–36).

Reference:

Leung, F. K.S. (2001). In search of an East Asian identity in mathematics education. *Educational Studies in Mathematics*, 47(1), 35-41.

Is the East Asian stereotype an accurate guide to the teaching of mathematics in Singapore schools?

The CORE 2 Study in Singapore

A Study of Pedagogical Practices in Grade 9 Mathematics and English Language

(Source of data presented in the following
slides is from Hogan et al., (2013))

The CORE 2 Study in Singapore

A Study of Pedagogical Practices in Grade 9 Mathematics and English Language

- The data reported here is from a nationally representative sample of over 4000 grade 9 students in approximately 120 mathematics and English classes across 32 secondary schools in Singapore collected in 2010.
- A split-half multi-level sampling strategy was used. In each class half of the students were randomly assigned to a **230-item survey focused on students' perceptions of instructional practices in mathematics or English Language.**
- In this presentation we focus on the four models of instruction explored in the study.
- The **models** are:
 - Traditional Instruction (**TI**)
 - Direct Instruction (**DI**)
 - Teaching for Understanding (**TfU**)
 - Co-Regulated Learning Strategies (**CRLS**)

Models of Instruction (Hogan et al., 2013)

Traditional Instruction (TI) (5 constructs)

1. **a focus on worksheets and workbooks** ('How often does your mathematics/English teacher ask you to do worksheets or workbooks?');
2. **a focus on textbooks** (e.g., 'How often does your mathematics teacher asks you to answer questions from the textbook?');
3. **drill and practice of basic facts, rules and procedures** (e.g., 'How often does your mathematics/English teacher ask you to drill and practice on basic facts, rules or procedures?');
4. **a focus on memorization** (e.g., 'How often does your mathematics teacher ask you to remember formulae or rules?'); and
5. **exam preparation** ('my teacher emphasizes studying problems that may occur in the exams'; 'my teacher spends a lot of class time preparing for exams'; 'my teacher teaches us test-taking strategies'; and 'my teacher emphasizes practicing past year exam papers').

Direct Instruction (DI) – 5 constructs

- **maximum learning time** (e.g., ‘The teacher makes sure that pupils focus on the lesson’);
- **teacher revision** (e.g., ‘The teacher checks that pupils understand the lesson’);
- **structure and clarity** (e.g., ‘The teacher clearly states the objectives of the lesson’, ‘The teacher organizes information in an orderly way’, ‘The teacher explains things very clearly’);
- **frequency of practice** (e.g., ‘We spend a lot of time practicing what we learned’); and
- **frequency of questioning** (e.g., ‘The teacher asks the class lots of questions’).

Teaching for Understanding (TfU) – 11 constructs

- **focus on understanding** (e.g., ‘The teacher’s explanations really help me understand the topic’).
- **quality of questions** (e.g., ‘The teacher asks good questions to see if we really understand’).
- **communicating learning goals and performance standards** (e.g., ‘The teacher explains the standard of good performance in our tests and exams’).
- **curiosity and interest** (e.g., ‘The teacher makes mathematics/ English really interesting’).
- **flexible teaching** (e.g., ‘The teacher tries different kinds of teaching to help us understand better’).
- **whole class discussion** (e.g., ‘The teacher supports long class discussions about topics’).
- **collaborative group work** (e.g., ‘The teacher encourages students to work as a team in group work’).
- **teacher scaffolding of group work** (e.g., ‘The teacher shows us how to work together in groups’).
- **monitoring of student learning** (e.g., ‘The teacher asks the class questions to see how well we understand the topic at the beginning of the class’).
- **personal feedback** (e.g., ‘The teacher gives me personal comments on my homework’).
- **collective feedback** (e.g., ‘The teacher gives the class detailed comments on exams or tests’).

Co-Regulated Learning Strategies (CRLS) comprises three multi-item first order scales for :

self-directed learning

The teacher encourages us to

- set our own learning goals;
- identify strategies to achieve our learning goals;
- check frequently that our work is acceptable

self-assessment

The teacher

- asks us to grade our own work;
- explains how we can grade our own work;
- expects us to discuss our own grading of our own work
- encourages us to comment on our own work.

peer-assessment

The teacher

- asks students to grade each other's work
- explains how we can grade each other's work
- expects us to discuss our grading of each other's work
- encourages us to comment on each other's work.

Findings (Hogan et al., 2013)

Traditional Instruction

	Mathematics	
	Mean (1-5)	SD
N	1166	
Traditional Instruction (TI) Scale (alpha: .758, .726)	3.78	.65
Focus on memorization	4.06	.91
Focus on worksheets and workbooks	3.93	.95
Focus on textbooks	3.83	.82
Focus on drill and practice on basic facts, rules and procedures	3.59	.97
Focus on exam preparation	3.51	.86

Direct Instruction

	Mathematics	
	Mean (1-5)	SD
N	1166	
Direct instruction (alpha: .844, .850)	3.61	.668
Maximum learning time (4)	3.89	.767
Structure and clarity (6)	3.61	.812
Teacher revision (4)	3.59	.835
Frequency of practice (1)	3.49	.952
Frequency of questioning (1)	3.47	.946

Teaching for Understanding

	Mathematics	
	Mean (1-5)	SD
N	1166	
Teaching for Understanding scale	3.38	.602
Collective feedback	3.59	.805
Communicating learning goals and performance standards	3.57	.771
Flexible teaching	3.57	.873
Monitoring student learning	3.46	.801
Personal feedback	3.43	.829
Focus on learning (understanding)	3.36	.710
Quality of questioning	3.34	.790
Curiosity and interest	3.25	.898
Whole class discussion	2.97	1.040
Collaborative group work	2.87	.962
Teacher scaffolding of group work	2.79	1.023

Co-Regulated Learning Strategies

	Mathematics	
	Mean (1-5)	SD
N	1166	
Co-regulated learning strategies (Alpha = .918, .920)	3.01	.770
Self-directed learning	3.41	.794
Self-assessment	2.92	.907
Peer assessment	2.80	.945

Summary of Findings

	Secondary 3 mathematics	
	Mean (1–5)	SD
TI	3.69	.642
DI	3.67	.670
TfU	3.38	.602
CRLS	3.01	.770

Although the strength of TI might lead one to conclude that mathematics instruction at least conforms to the **East Asian Stereotype**, the relative strengths of the other instructional strategies suggest otherwise.

This conclusion is supported by the high correlations between DI, TI and TfU. The substantially lower correlations between TI, and DI with CRLS explains the active instructional role of the teacher in the classroom.

Correlation matrix: instructional methods Secondary 3 mathematics :

	TI	DI	TfU	CRLS
<i>Mathematics</i>				
Traditional instruction	1			
Direct instruction	.72**	1		
Teaching for understanding	.58**	.70**	1	
Co-regulated learning strategies	.28**	.35**	.73**	1

Source: Hogan et al., (2013)

SEM models and what they tell us

SEM model for Traditional Instruction and Direct Instruction

- Outcome variable – frequency of questioning (memorization did not give the best fitting model)
- Internal structure of the two sets of instructional strategies remained remarkably stable in the model.
- All the TI constructs had pathways leading to DI constructs.
- The density and strength of the pathways cast considerable doubt on TI or DI as discrete instructional categories; it is best viewed as constituting an integrated, theoretically meaningful hybridic model of instructional practice.

Note: The SEM models are reported in detail in Hogan et al., (2013). Assessment and the logic of instructional practice in Secondary 3 English and mathematics classrooms in Singapore. *Review of Education*, 1(1), 57-106.

SEM model for Teaching for Understanding and Co-Regulated Learning Strategies

- Outcome variable – focus on learning (gives the best fitting model). It also indexes the degree to which teachers focus on their instruction on meaning making and developing student understanding.
- The model has exceptional good fit , rich, sensible and suggestive networks of pathways, strong coefficients, and theoretical gravitas.

Note: The SEM models are reported in detail in Hogan et al., (2013). Assessment and the logic of instructional practice in Secondary 3 English and mathematics classrooms in Singapore. *Review of Education*, 1(1), 57-106.

Integrated SEM model for all four instructional strategies

- This model exemplifies the overall structure of the 4 instructional strategies jointly at the construct level in an integrated model.
- Bearing in mind that the model is very large and complex, the goodness-of-fit statistics are exceptionally good.
- The model is fully recursive – there are no feedback loops from TfU back into TI or DI practices.
- The internal structure of each of the instructional strategies remained remarkably stable.
- There is a linear, fully recursive sequence to instructional practice that underscores the coherent and hybridic nature of the instructional regime for mathematics in Singapore grade 9 classrooms.

Note: The SEM models are reported in detail in Hogan et al., (2013). Assessment and the logic of instructional practice in Secondary 3 English and mathematics classrooms in Singapore. *Review of Education*, 1(1), 57-106.

Our conjecture!

Instructional practices for mathematics in Singapore classrooms, based on the data of the CORE 2 study, cannot be considered either Eastern or Western but a coherent combination of both.

Why?

- **TI** provides the foundation of the instructional order.
- **DI** builds on TI practices and extends and refines the instructional repertoire, while TfU/CRLS practices build on TI and DI practices and extend the instructional repertoire even further in ways that focus on developing student understanding and student directed learning.

Source: Hogan et al., 2013

What then ties or links the four instructional groupings together in an orderly chain of instructional practice?

- Four instructional practices - two TI practices (exam preparation and textbook focus) and two DI practices (structure and clarity, and revision).
- Of the four, exam preparation is the most significant. It is highly generative both directly and indirectly, reaching well beyond its own close family of TI practices into DI and TfU practices.
- There are nine separate direct pathways leading from exam preparation to DI and TfU practices, and numerous indirect paths that link exam preparation, on the one hand, to all of the remaining instructional practices, on the other.

Source: Hogan et al., 2013



What's next!



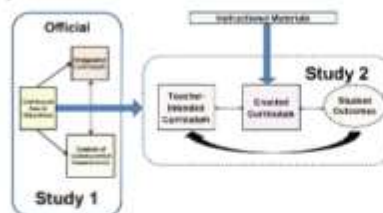
MME PROGRAMMATIC RESEARCH PROJECT

A Study of the Enacted School Mathematics Curriculum (Secondary)

(Project No: OER 31/15BK)

Aim:

To study the enacted school mathematics curriculum in the secondary school.



Study 1:

Pedagogies adopted by experienced mathematics teachers when enacting the curriculum.

- How do teachers introduce and engage students in constructing conceptual knowledge?
- How do teachers engage students in developing fluency with skills in computing?
- What mathematical processes are used and developed by teachers?
- How do teachers imbue desired attitudes for the learning of mathematics amongst their students?

Study 2:

Experienced secondary school mathematics teachers' use of instructional materials for the enactment of the curriculum.

- How do teachers select instructional materials?
- How do teachers modify the selected instructional materials?
- What are the characteristics of "instructional materials" that will
 - help teachers enact worthy instructional goals of teaching mathematics and
 - help students achieve desirable outcomes?



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THANK YOU

