

**THE COORDINATION BETWEEN THE GEOMETRIC FIGURE AND THE ASSOCIATED
MAGNITUDE AND MEASURE, WITHIN THE CONCEPTUAL GENESIS OF ANGLE**

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The starting point of the proposed reflections is a classroom activity using *GeoGebra* (Tanguay et al., 2013), intended for the first year of secondary schooling. The activity falls within a larger research program about teaching Geometry, based on what we called the geometer-physicist paradigm (Tanguay & Geeraerts, 2012). This framework, inspired by Jahnke's work (e.g. 2007), is aimed at reconciling empirical and deductive processes, by among other things assessing measurements with greater care, giving them back a role scientifically better assumed and controlled.

Firstly, we'll analyze the work of two 12 years old students during the first phase of the above-mentioned activity. In the course of their actions and verbal exchanges, these students gave manifest clues to their conceptions and representations of angle, and to the influence that measuring tasks, recurrent at the primary level, may have had on these conceptions. Indeed among the notions of elementary geometry, angle is without a doubt the most difficult to conceptualize: is it a figure or a measure? In the first case, what would be the magnitude gauged by angle measurement? In the second case, of which geometrical object the measure at issue is an attribute?

To better understand the conceptual and semiotic geneses of angle hinted by this experiment, we submitted a questionnaire about angles to a class of students in the last year of their elementary schooling (11-12 years old), and to groups of students in a preservice elementary teacher training program. The semiotic genesis that the answers suggest is analyzed in terms of sense and meaning (in the vygotskian perspective associated with these words), and makes also use of Arzarello's notion of *semiotic bundle*. We call into question the process of sense construction by amalgamation proposed by Arzarello (2006), to put forward instead a dialectical process of amalgamation-disentanglement (Tanguay, 2015). This process, as we will argue, allows to better account for students' difficulties in adequately conceiving of angle.

References

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